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SOME RESULT FOR BANACH'S FIXED POINT THEOREM

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ABSTRACT

In the present paper a new idea has been established for fixed point theorem for a self mapping which satisfying contraction mapping on a complete matrix space in Banach space.

KEYWORDS: Fixed Point, Banach Space, Self Mapping, Contraction Mapping, Non Expansive Mapping

Today with the advancement of science in Mathematical era fixed point theory in Banach space of contraction mappings is well establishing and their existence is define in a large scale in the field of mathematics with new amplitude. Lipschitz in the area of fixed point it is a type of contractive mapping role plays an important role in Banach space. It is obvious that the Banach (1622) contraction principle is a fundamental result in fixed point theory, which used in different direction of matrix space. There are many mathematicians such as Bose and Mukherjee (1981), De marr (1963), Browder (1965) who defined some useful properties of the fixed point for contractive mapping in Banach spaces. A mapping sometimes not may be non-expansive but when it composite with other mapping it will be non-expansive, so the composite as well as it's sequences have a common fixed point. Dotson and Mann (1977), Jungck (1976) studied some fixed point theorem for non-expansive mappings in Banach spaces.

(i) Suppose F is a bounded convex subset of a banach space β and $S: F \to F$ is a mapping, where β is a non-expansive if:

$\|S(u) - S(v)\| \leq \|u - v\| \quad \forall\, u, v \in F.$ (1.a)

Let $G(S) = \{u \in F: S(u) = u\}$, where $G(S)$ is called a set of fixed point for mapping S. If F is a closed and convex subset of a Hilbert space h and T has a fixed point, then $\forall u \in F, \{S_n(u)\}$ is convergent to a fixed point of S. This theorem was proved by Baillob (1975) in Hilbart space for non-expansive mapping. The mapping S is known as quasi-non expansive mapping by Shahzad (2004) if:

$\|S(u) - e\| \leq \|u - e\|$ for all $x \in F, e \in G$ (1.b)

THE BASIC RESULTS

Theorem (a)

Let S be a self mapping for a non-empty,compact and convex subset of a Banach space β which commutes S_μ where $0 < \mu < 1$ and t is a fixed number of convex subset then

$\|SS_\mu(u) - SS_\mu(v)\| \leq \|u - v\|$ (c)

where u, v are fixed elements of convex subset of Banach space β

Proof

As we know that SS_μ is nonexpansive and mapping a convex subset into itself then SS_μ has a fixed point k in convex subset then,

$\|S(k) - k\| = \|SSS_\mu(k) - SS_\mu(k)\|$

$= \|S_\mu S^2(k) - S_\mu S(k)\|$

$= \mu\|S^3(k) - S^2(k)\|$ …(d)

Similarly $\|S^3(k) - S^2(k)\| = \mu\|S^5(k) - S^4(k)\|$

Thus from equation (d) $\|s(k) - k\| = \mu^2\|S^5(k) - S^4(k)\|$

So, in general we have

$\|s(k) - k\| = \mu^p\|S^{2p+1}(k) - S^{2p}(k)\|$

where $p = 1,2,3, \ldots$ …(e)

Since sub convex set is compact so we can obtain a positive number δ such that

$\|s(k) - k\| \leq \delta\mu^p$ …(f)

If $\mu < 1, p \to \infty$ we have $S(k) = k$

Therefore k is a common fixed point of S, S_μ and SS_μ

(ii) For fixed point Fisher (1979) take a mapping $f: U \to U$ satisfying the conditionof type

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$[d(fu,fv)]^2 \leq \alpha d(\mathrm{u},\mathrm{fu})\mathrm{d}(\mathrm{v},\mathrm{fv}) + \beta \mathrm{d}(\mathrm{u},\mathrm{fv})\mathrm{d}(\mathrm{v},\mathrm{fu})$ .... (g)

For all $u, v \in X$ and $0 \leq \alpha < 1$ and $\beta \geq 0$

George *et al.*, (2017) gives application of many fixed point theorem

Theorem (b)

Let U be a self mapping of a non-empty compact and convex subset of a Banach β space and $S_\mu(u) = \mu S(u) + (1-\mu)u$, where$0 < \mu < 1$. If S and S_μ commute for all $u, v \in C$, so S, S_μ and SS_μ have a common fixed point in C.

Proof

Let k be a fixed point of SS_μ then

$k = SS_\mu(k) = \mu S^2(k) + (1-\mu)S(k)$

Then,$\|S(k) - S\| = \mu\|S^2(k) - S(k)\|$ ...(h)

Also, $S(k) = S\left(S_\mu(k)\right) = S_\mu S^2(k) = \mu S^3(k) + (1-\mu)S^2(k)$

That is, $\|S^2(k) = S(k)\| = \mu\|S^3(k) - S^2(k)\|$ ...(i)

So from above we have,

$\|S(k) - S)\| = \mu\|S^3(k) - S^2(k)\|$

Thus in general

$\|S(k) - S\| = \mu^p\|S^{p+1}(k) - S^p(k)\|$, where p =1,2,3...

When p tending to infinite, we get

$S(k) = k$

So this is the desired results.

Theorem (c)

Let U be a closed and convex subset of a Banach space and u, v be any two points of U and f is a contraction on U then for a positive integer n satisfy the condition

$\mathrm{d}\left(\mathrm{f}^{\mathrm{n}}(\mathrm{u}), \mathrm{f}^{\mathrm{n}}(\mathrm{v})\right) \leq \alpha^{\mathrm{n}}\mathrm{d}(\mathrm{u},\mathrm{v})\forall \mathrm{u},\mathrm{v} \in \mathrm{U}$, where $0 \leq \alpha < 1$

Proof

Let u_0 be any point of U and let m, n be two positive integer greater than 1 then we have

$$d(u_n, u_m) = d(u_n, u_{n+p}) \leq d(u_n, u_{n+1}) + d(u_{n+1}, u_{n+2}) + \cdots + d(u_{n+p-1}, u_{n+p})$$

$\mathrm{d}(f^n(u_0), f^n(u_1)) + d(f^{n+1}(u_0), f^{n+1}(u_1)) + \cdots + d(f^{n+p-1}(u_0), f^{n+p-1}(u_1))$ ...(j)

with the help of (g) and (j),we have

$d(u_m, u_n) \leq \alpha^n d(u_0, u_1) + \alpha^{n+1} d(u_0, u_1) + \cdots + \alpha^{n+p-1} d(u_0, u_1)$

$$= \frac{\alpha^n d(u_0, u_1)}{1-\alpha}$$

Since $\lim_{n\to\infty} \alpha^n = 0$ this implies that $d(u_n, u_m)$ can be made less than any pre-assigned positive number $\in, so\ t\square at\ b \lim_{n\to\infty} f(u_n) = f(u) = u$ because $\langle f(u_n)\rangle$ converge to u

Hence $f(u) = u$

This shows that u is a fixed point

Hence proved the theorem.

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REFERENCES

Banach S., 1622. Sur les operations dans less ensembles abstraites et leurs applications, Fund. Math., **3**:133-181.

Bose R.K. and Mukherjee R.N., 1981. Approximating fixed points of some mapping, proc. Amer. Math. Soc., **82**:603-606.

Browder F.E., 1965. Nonexpansive nonlinear operators in a Banach space, Proc. Nat. Acad. Sci., U.S.A., **54**:1041-1044.

Baillon J.B., 1975. Un theorem de type ergodique pourles contractions non lineaies dans un espace de

Hilbert,'' Comptes Rendus de 1' Academic des Sciences de Paris, Serie A 280, **22**:1511-1514.

DeMarr R., 1963. Common fixed points for commuting contraction mappings, Pacific J. Math., **13**:1139-1141.

Dotson W.G. and Mann W.R., 1977. The Schauder fixed point theorem for non-expansive mappings, amer. Math. Monthly, **84**:363-364.

Fisher B., 1979. Mappings with a common fixed point,R Raj Math. Sem. Notes, Kobe Univ., **7**:81-84.

George R.A., Nabwey H. and Rajagoplan, 2017. Rectangular cone ang b- metrix space over Banach algebra and contraction principle, Fixed point and its application, **14**.

Jungck G., 1976. Commuting mappings and fixed points, Amer. Math. Montly, **83**:261-263.

Shahzad N., 2004. "Generalized I-nonexpansive maps and best approximations in Banach spaces", demonstratio Mathematics, **37**(3): 597-600.